impedance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 259–265, May 1969.

[7] H. A. Wheeler, "Transmission-line properties of a strip line between parallel planes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 866–876, Nov 1978.

[8] S. Kusase and R. Terakado, "Mapping theory of two-dimensional anisotropic regions," *Proc. IEEE*, vol. 67, Jan. 1979, pp. 171–172.

[9] F. Bowman, *Introduction to Elliptic Functions with Applications.* New York: Dover, 1961, pp. 71–72.

[10] J. Fontanella, C. Andeen, and D. Schuele, "Low-frequency dielectric constants of α-quartz, sapphire, MgF_2 and MgO," *J. Appl. Phys.*, vol. 45, pp. 2852–2854, July 1974.

[11] H. J. Riblet, "Two limiting values of the capacitance of symmetrical rectangular coaxial strip transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 661–666, July 1981.

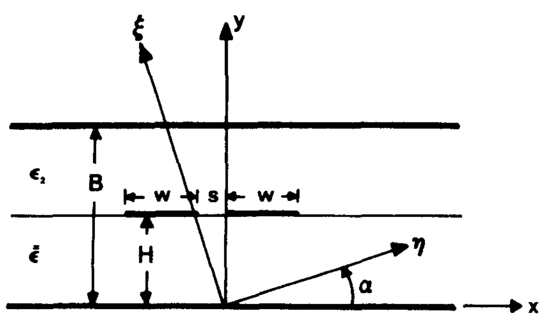

Fig. 1. Microstrip geometry showing crystal (ξ,η) and microstrip (x, y) axis.

Characteristics of Microstrip Directional Couplers on Anisotropic Substrates

NICOLAOS G. ALEXOPOULOS, SENIOR MEMBER, IEEE, AND S.A. MAAS

Abstract —**The properties of directional couplers fabricated on anisotropic substrates are examined on a quasistatic basis. Three substrate materials are considered: sapphire, Epsilam (a ceramic-filled polytetrafluoroethylene dielectric), and boron nitride. VSWR, directivity, and coupling are presented for several representative 10-dB coupler designs, as well as variations in these parameters with crystal axis offsets. It is demonstrated that high directivity can be achieved by making use of the substrate anisotropy in conjunction with a top cover to equalize even- and odd-mode phase velocities.**

I. INTRODUCTION

The need to improve the performance repeatability of microwave hybrid integrated circuits has generated considerable interest in the use of crystalline substrate materials such as sapphire and boron nitride with highly predictable dielectric properties. At the same time, the need for low cost microstrip circuits has generated a number of glass- and ceramic-filled polymeric materials, by such trade names as Duroid and Epsilam. These materials are electrically anisotropic either because of their crystalline structure or the processes used in their manufacture. In fact, it has been demonstrated that other, presumably isotropic materials such as alumina, show significant process-dependent anisotropy.

Anisotropy in microstrip substrates has traditionally been regarded as an undesirable property, primarily because microstrip impedances and phase velocities are somewhat more difficult to determine for anisotropic than for isotropic materials. However, in the case of directional couplers and other coupled-line components, certain types of anisotropy can be of significant advantage. The odd-mode phase velocity in a microstrip coupler using an isotropic substrate is greater than the even-mode velocity causing degradation of the coupler's directivity. This degradation becomes worse as the coupling is decreased or as the dielectric permittivity is increased. Furthermore, the effects of unmatched phase velocities are more severe than the effects of incorrect even- and odd-mode impedances. For example, a 10 percent difference in phase velocities reduces the directivity of a 10-dB coupler to 13 dB from its theoretically infinite value with equal phase velocities. On the other hand, a 10 percent error in either even- or odd-mode impedance, with equal phase velocities, reduces the directivity only to 26 dB. The corresponding figures for a 20-dB coupler are 2 dB and 26 dB, respectively. Invariably, the even-mode phase velocity is less than the odd-mode velocity by 10–12 percent for a 10-dB coupler, depending on the permittivity of the dielectric substrate.

However, if the substrate is anisotropic and the permittivity in a direction parallel to the ground plane is greater than the perpendicular component [1], the odd-mode phase velocity will be reduced relative to the even mode and significantly improved directivity will result [2]. To equalize the phase velocities, the parallel component (the $x-z$ component of Fig. 1) must be approximately twice the perpendicular y-component [1],[2]. This degree of anisotropy is virtually nonexistent in practical microwave substrate materials, although a factor of 1.2–1.5 has been observed. Nevertheless, these practically achievable anisotropies can be used to significantly improve coupler directivity, and changes in the microstrip geometry—most notably in the top cover height for shielded microstrip—can be used in conjunction with anisotropy to equalize even- and odd-mode phase velocities.

II. ANALYSIS

Anisotropic materials have received recently considerable attention [3]–[7] as microstrip substrates. In this article, the anisotropy is described by a tensor relative permittivity given by

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}. \tag{1}$$

If the axes of the substrate are aligned with the axes of the crystal (see Fig. 1) then $\epsilon_{yx} = \epsilon_{xy} = 0$. If, however, the alignment is imperfect, the elements of $\bar{\bar{\epsilon}}$ above are given by

$$\epsilon_{xx} = \epsilon_{\xi\xi} \cos^2\alpha + \epsilon_{\eta\eta} \sin^2\alpha \tag{2}$$

$$\epsilon_{yy} = \epsilon_{\xi\xi} \sin^2\alpha + \epsilon_{\eta\eta} \cos^2\alpha \tag{3}$$

and

$$\epsilon_{xy} = \epsilon_{yx} = (\epsilon_{\xi\xi} - \epsilon_{\eta\eta}) \sin\alpha \cos\alpha \tag{4}$$

where the ξ,η subscripts refer to the crystal axes. As discussed in [8], a quasi-static analysis of the problem is sufficient for lower frequencies. In order to compute the characteristic impedance and phase velocity of the coupled lines, an even–odd mode

Manuscript received December 29, 1981; revised March 2, 1982. This work was supported in part by U.S. Army Contract DAAG-29-79-C-0050.

N. G. Alexopoulos is with the Electrical Engineering Department, University of California, Los Angeles, CA 90024.

S. A. Maas is with TRW, Redondo Beach, CA 90278.

0018-9480/82/0800-1267$00.75 ©1982 IEEE

approach is used in combination with the method of moments [1],[9]. To this end a quasi-TEM approach is sought by solving Laplace's equation

$$\nabla \cdot \left[\bar{\bar{\epsilon}}_j \cdot \nabla G_j \right] = 0 \tag{5}$$

with $j = 1,2$ pertaining to the region of interest (e.g., for $j = 1, \bar{\bar{\epsilon}}_1 = \bar{\bar{\epsilon}}$ is given by (1) and for $j = 2, \bar{\bar{\epsilon}}_2 = \epsilon_2 = 1$). The boundary conditions are simply

$$G_1(x,x';y,y')|_{y=y'=0} = G_2(x,x';y,y')|_{y=y'=B} = 0 \tag{6}$$

$$G_1(x,x';y,y')|_{y=y'=H} = G_2(x,x';y,y')|_{y=y'=H} \equiv G(x,x') \tag{7}$$

$$\hat{y} \cdot \left[\bar{\bar{\epsilon}} \cdot \nabla G_1(x,x';y,y') - \nabla G_2(x,x';y,y') \right]_{y=y'=H} = \frac{\delta(x)}{\epsilon_0}. \tag{8}$$

The solution to the above boundary value problem can be obtained by considering

$$G(x,y) = \mathrm{Re}[\phi(x,y)] \tag{9}$$

and the spectral representation

$$\phi(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(\xi,y) e^{+j\xi x} d\xi \tag{10}$$

with

$$\psi(\xi,y) = \int_{-\infty}^{\infty} \phi(x,y) e^{-j\xi x} dx. \tag{11}$$

On the basis of this consideration, the Green's function to this problem is written as

$$G(x,x') = \frac{1}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{\cos[\lambda|x-x'|]\, d\lambda}{\lambda[\gamma\delta \coth(\lambda\delta H) + \epsilon_2 \coth(\lambda H \nu)]} \tag{12}$$

with

$$\nu = \frac{B}{H} - 1, \gamma = \epsilon_{yy} \qquad \delta = \left[\epsilon_{xx}/\epsilon_{yy} - (\epsilon_{xy}/\epsilon_{yy})^2 \right]^{1/2}.$$

Analytic continuation and the Cauchy Residue Theorem convert (5) into a series form

$$G(x,x') = \frac{1}{\epsilon_0 H} \sum_{p=1}^{\infty} \frac{\exp[-v_p|x-x'|]}{v_p\left[\gamma\delta^2 \csc^2(v_p\delta H) + \epsilon_2 \nu \csc^2(v_p \nu H)\right]} \tag{13}$$

where the v_p is the pth root of

$$\sin(v_p\Sigma) = -\left[\frac{\left(\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}^2\right)^{1/2} - \epsilon_2}{\left(\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}^2\right)^{1/2} + \epsilon_2} \right] \sin(v_p\Delta) \tag{14}$$

and

$$\Sigma = \nu + \delta, \Delta = \nu - \delta. \tag{15}$$

Both Green's function representations have been utilized and the results obtained have been compared with various data in the literature, notably with the isotropic substrate results of Bryant and Weiss [10]. Although both Green's function representations (i.e., (5) and (6)) give highly accurate phase velocities, agreeing with [10] to 1 percent or better, the series solution is consistently 1 percent–2 percent low in impedance. The integral expression however, is remarkably accurate, with agreement within 0.5 percent for practical geometries. Although the series solution requires approximately 1/3 the time, all data in this paper were determined by using the more accurate integral form given by (12).

III. Coupler Designs

Several designs of couplers were considered for anisotropic substrates and compared with cases where the substrate is isotropic. The anisotropic materials adopted in these designs were Epsilam-10, sapphire, and boron nitride. In all cases a single-section 10-dB coupler was used for comparison.

Epsilam is a ceramic-filled PTFE material. Its anisotropy arises from its manufacturing process, and as a result the extraordinary axis of the material is inherently aligned with the microstrip y-axis. The $x-z$ relative permittivity of the old (still in production) Epsilam material is approximately 13.5 for 0.025-in thick material; the y relative permittivity is $\simeq 10$. Since it is a composite material, these parameters tend to vary between manufacturing lot, e.g., 0.05-in thick material has an $x-z$ permittivity of $\simeq 15$.

Boron nitride has $x-z$ and y relative permittivities of 5.15 and 3.4, respectively, the strongest anisotropy of all three materials considered. It has a low-loss tangent as well, and is potentially a useful material for microstrip applications. It is not as yet, however, in wide use.

Sapphire has been widely accepted for use in high frequency, low-loss microwave hybrids. However, its $x-y$ relative permittivity of 9.4 and y relative permittivity of 11.6 are the opposite of those desired for high coupler directivity. Sapphire could conceivably be supplied with the 11.6 direction parallel to the microstrip ground plane, and the 9.4 perpendicular, but then the axis of the coupler would have to be aligned with the z-axis of the crystal. This would pose a serious layout problem in many circuits. It is important to recognize that couplers designed on conventionally-cut sapphire will have significantly worse directivity than those fabricated on isotropic materials.

Coupler performance is summarized in Table I. For each design, S/H and w/H were adjusted until the desired Z_{0o} and Z_{0e} of 36.0 and 69.4 Ω for a 10-dB coupler in a 50 − Ω system were achieved. In addition, for the shielded couplers, the normalized top cover height B/H was adjusted until v_{po} and v_{pe} were equal. This is a laborious and expensive process, and it was therefore terminated when impedances and phase velocities were within about 1 percent of the desired values. The unshielded coupler was approximated as a shielded coupler with a top cover height of $B/H = 6$; for $B/H > 6$, the top cover has no significant effect on microstrip parameters [1],[9].

Table I shows a directivity of 12–13 dB for a coupler fabricated on an isotropic substrate. Using an anisotropic substrate such as Epsilam 10 (0.025 in thick) results in 6-dB improvement in directivity, to 19 dB. The unshielded coupler on boron nitride has 19.5-dB directivity. Of course, if impedances are exact and phase velocities are equal, directivity is theoretically infinite. Table I shows extremely high directivity for Epsilam and boron nitride couplers with approximately equal even- and odd-mode phase velocities. In practice, the directivities of such couplers will be limited by such things as manufacturing tolerances and reflections at transitions into and out of the coupled line segments. With care, however, directivities greater than $\simeq 20$ dB below 10 GHz should be achievable. This is considerably better than the 10–12-dB directivity usually achieved for 10-dB couplers on isotropic substrates.

Fig. 2 shows the variation in even- and odd-mode parameters for boron nitride as the crystal axes are rotated with respect to the microstrip axes: this situation would be presented if the

TABLE I
COUPLER DESIGN—SUMMARY

COUPLER	w/H	B/H	S/H	Z_{oe}	Z_{oo}	(10^8 m/s) v_{pe}	(10^8 m/s) v_{po}	DIREC-TIVITY dB	VSWR	CENTER FREQ. COUPLING
EPSILAM - SHIELDED	0.700	2.55	0.260	69.0	35.9	1.207	1.210	43	< 1.01	10.02
EPSILAM - UNSHIELDED	0.800	>6	0.280	69.4	36.0	1.138	1.204	18	1.03	10.00
ALUMINA UNSHIELDED	0.875	>6	0.260	69.2	35.9	1.150	1.286	12	1.06	10.04
BORON NITRIDE - UNSHIELDED	1.850	>6	0.120	70.0	35.9	1.772	1.860	19.5	1.03	9.83
BORON NITRIDE - SHIELDED	1.60	2.80	0.095	69.3	36.0	1.876	1.875	58	< 1.01	10.00
QUARTZ - UNSHIELDED	1.830	>6	0.110	69.2	36.2	1.708	1.886	13	1.05	10.13
SAPPHIRE - SHIELDED, 90° OFFSET (ϵ_{xx}=11.6, ϵ_{yy}= 9.4)	0.690	2.20	0.225	69.2	35.9	1.256	1.257	49	1.01	9.98
SAPPHIRE - UNSHIELDED (ϵ_{xx}= 9.4, ϵ_{yy}= 11.6	0.730	>6	0.260	69.4	36.2	1.086	1.227	11	1.06	10.12

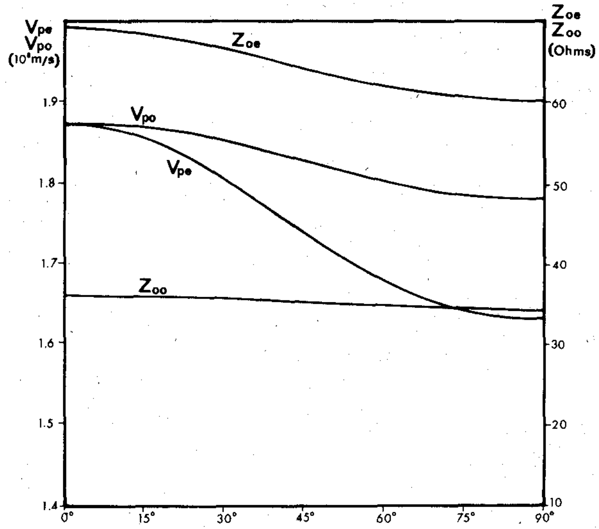

Fig. 2. Boron nitride 10-dB coupler. Coupled-line parameters versus crystal axis rotation α. $S/H = 0.095$, $W/H = 1.6$, $\epsilon_{xx} = 5.12$, $\epsilon_{yy} = 3.4$ at $\alpha = 0°$.

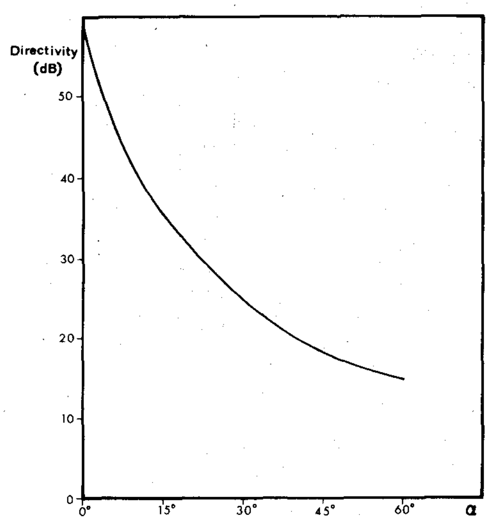

Fig. 3. 10-dB coupler boron nitride substrate directivity versus rotation α. $B/H = 2.8$, $S/H = 0.095$ $w/H = 1.6$.

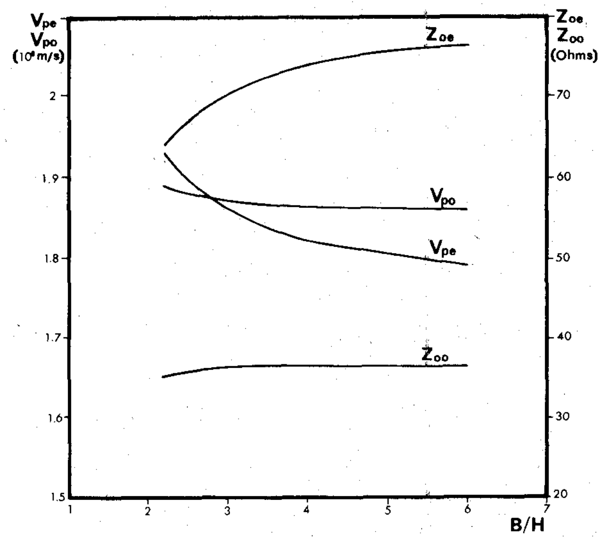

Fig. 4. 10-dB coupler boron nitride substrate microstrip parameters versus B/H. $w/H = 1.6$, $S/H = 0.095$, $\epsilon_{xx} = 5$, $\epsilon_{yy} - 3.4$.

crystal were not cut precisely as desired. The resulting degradation in directivity is shown in Fig. 3. Fig. 4 shows the effect of changing the top cover height; it is clear that the phase velocities are only moderately sensitive to changes in B/H. The even-mode impedance is somewhat more sensitive in terms of percentage change for a specified change in cover height, but directivity is considerably less sensitive to errors in impedance than in phase velocity.

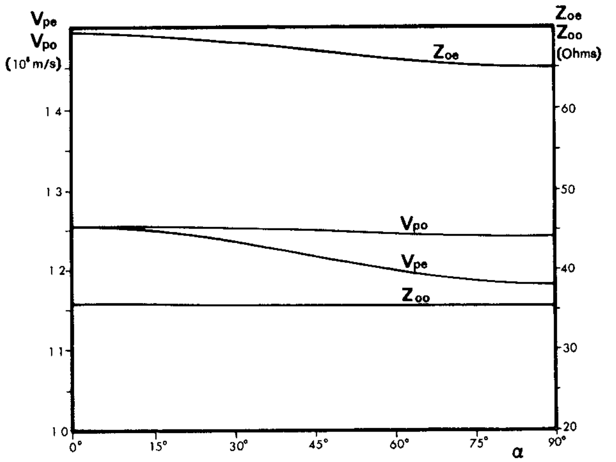

Fig. 5. 10-dB coupler, sapphire substrate. Coupled-line parameters versus rotation α. $\epsilon_{xx} = 11.6$, $\epsilon_{yy} = 9.4$ at $\alpha = 0°$, $B/H = 2.2$, $S/H = 0.225$, $w/H = 0.69$.

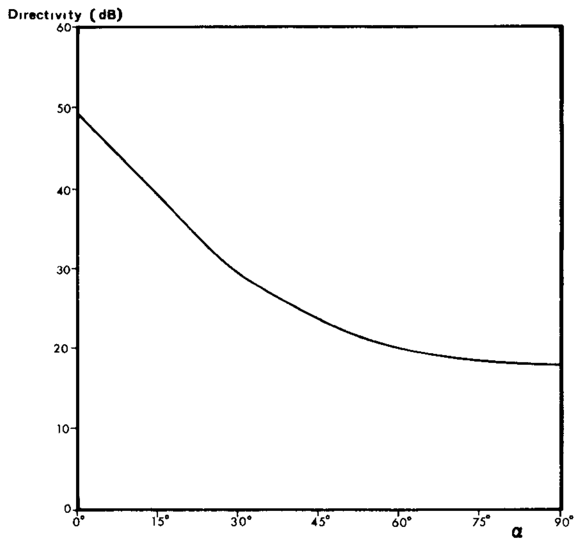

Fig. 6. 10-dB coupler, sapphire substrate. Directivity versus rotation α. $B/H = 2.2$, $S/H = 0.225$, $w/H = 0.69$, $\epsilon_{xx} = 11.6$, $\epsilon_{yy} = 9.4$ at $\alpha = 0°$).

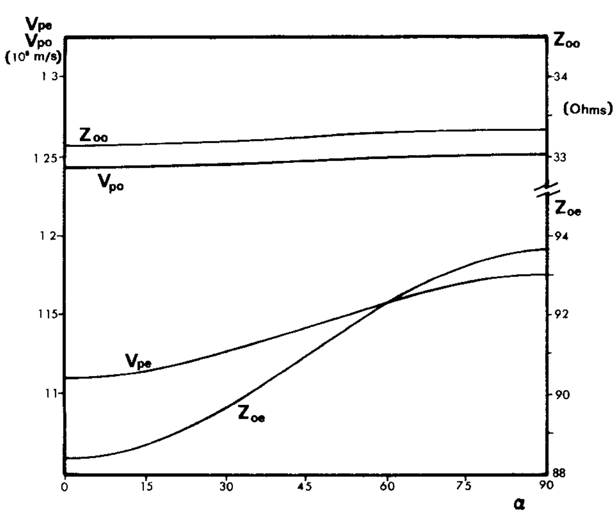

Fig. 7. 10-dB coupler, sapphire substrate. Coupled-line parameters versus crystal axis rotation α. $S/H = 0.1$, $B/H = 6.0$, $\epsilon_{xx} = 9.4$, $\epsilon_{yy} = 11.6$ at $\alpha = 0°$.

Comparable data for sapphire is presented in Figs. 5–7. No data for Epsilam as a function of crystal axis offset is presented, as it is an inherently aligned material.

IV. Conclusions

It has been shown that performance of coupled-line components, especially directional couplers, can be improved by using the anisotropy of certain substrate materials to equalize even- and odd-mode phase velocities. This can be accomplished by choosing a material with higher permittivity parallel to the ground plane than perpendicular, and lowering the top cover height. Materials with strong anisotropies are particularly suitable for this technique, and boron nitride in particular is a promising material. Sapphire is a less promising material, and couplers fabricated on sapphire substrates as conventionally fabricated may have worse directivity than those on isotropic substrates. For a sapphire substrate, high directivity is obtained when $B/H \simeq 1.9$; however, the performance is extremely sensitive to small tolerances of deviation from $B = 1.9\ H$.

Acknowledgment

The authors wish to thank P. B. Katehi and L. Loberman for preparing the manuscript.

References

[1] N G Alexopoulos and C. M. Krowne, "Characteristics of single and coupled microstrips on anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 387–393, June 1978.

[2] N. G. Alexopoulos and N. K. Uzunoglu, "Characteristics of microstrip on anisotropic substrates," in *Proc. 7th Europ. Microwave Conf.*, Sept. 1977, pp. 140–143

[3] R. P. Owens, J. E. Aitken, and T C. Edwards, "Quasi-static characteristics of microstrip on an anisotropic sapphire substrate," *IEEE Trans Microwave Theory Tech.*, vol. MTT-24, pp. 499–505, Aug. 1976

[4] T. Edwards and R P. Owens, "2–18-GHz dispersion measurements on 10–100-Ω microstrip lines on sapphire," *IEEE Trans. Microwave Theory Tech.*, vol MTT-24, pp. 506–513, Aug 1976.

[5] N. G. Alexopoulos and N. K. Uzunoglu, "An efficient computation of thick microstrip properties on anisotropic substrates," *J. Franklin Inst.* vol. 306, no 1, pp 9–22, July 1978

[6] A. G. d'Assuncao, A. J. Giarola, and D. A Rogers, "Analysis of single and coupled striplines with anisotropic substrates," in *IEEE MTT-S Int Symp. Dig.*, June 1981, pp. 83–85

[7] A.M. A. El-Sherbiny, "Hybrid mode analysis of microstrip lines on anisotropic substrates," in *IEEE MTT-S Int. Symp. Dig.*, June 1981, pp 492–494.

[8] T. C. Edwards, *Foundations for Microstrip Circuit Design.* New York: Wiley, 1981.

[9] A Farrar and A. T. Adams, "Characteristic impedance of microstrip by the method of moments," *IEEE Trans. Microwave Theory Tech.*, vol MMT-18, pp. 65–66, Jan 1970.

[10] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol MTT-16, pp. 1021–1027, Dec. 1968.

Propagation of Picosecond Pulses on Microwave Striplines

K. K. LI, G ARJAVALINGAM, A. DIENES, AND J. R. WHINNERY, FELLOW, IEEE

Abstract —**Dispersion of picosecond pulses propagating on a microstrip line is calculated in the time domain. Numerical calculations are by the fast**

Manuscript received December 30, 1981; revised February 25, 1982. This work was sponsored in part by the National Science Foundation under Grant ENG 78-10233

K. K. Li, G. Arjavalingham, and J. R. Whinnery are with the Department of Electrical Engineering and Computer Sciences and the Electronics Research Laboratory, University of California, Berkeley, CA 94720.

A. Dienes is with the Department of Electrical Engineering and Computer Sciences and the Electronics Research Laboratory, University of California, Berkeley, CA 94720 He is also with the Department of Electrical and Computer Engineering, University of California, Davis, CA 95616.